

## New supersymmetric solutions in $N = 2$ matter coupled $AdS_3$ supergravities

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**ABSTRACT:** We construct new  $1/2$  supersymmetric solutions in  $D = 3, N = 2$ , matter coupled,  $U(1)$  gauged supergravities and study some of their properties. We do this by employing a quite general supersymmetry breaking condition, from which we also redrive some of the already known solutions. Among the new solutions, we have an explicit non-topological soliton for the non-compact sigma model, a locally flat solution for the compact sigma model and a string-like solution for both types of sigma models. The last one is smooth for the compact scalar manifold.

**KEYWORDS:** Solitons Monopoles and Instantons, Supergravity Models, AdS-CFT Correspondence.

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## 1. Introduction

Supersymmetric solutions in supergravity theories have been quite fundamental in understanding various aspects of string/M theory. However, such solutions were not studied much for supergravities coupled to non-linear sigma models due to their complexity. Because of its relative simplicity,  $D = 3$  supergravities provide a good framework for understanding such systems. This has the further advantage from the *AdS/CFT* perspective [1–3] since the dual theory would be a two dimensional *CFT*.

With these motivations in mind, we find new supersymmetric solutions in the matter coupled  $D = 3, N = 2$ ,  $U(1)$  gauged supergravities and study some of their properties. This model was constructed in [4] and admits both compact and non-compact sigma model manifolds. There is also a well-defined flat sigma model limit. The theory contains only a Chern-Simons gauge field and no Maxwell term. The first supersymmetric solutions of this model were constructed in [4] and these described static, uncharged strings. Later the charged, stationary generalizations of these strings superposed with gravitational and Chern-Simons electromagnetic waves were obtained in [5]. Another class of solutions representing vortices were found in [6], where the model we consider was modified with a Fayet-Iliopoulos term. This changes the potential so that only topological solitons, by which we mean smooth solutions that interpolate between AdS and Minkowski vacua, are allowed.

The supersymmetric vortices [6] and strings [5] were obtained by using structure-wise similar supersymmetry breaking conditions. In this paper, we consider a more general supersymmetry breaking condition which contains these previous cases and succeed in obtaining the Killing spinor explicitly. In addition to the already known solutions, this also leads us to new ones. Among these, we have an explicit non-topological soliton solution (a smooth solution that approaches to AdS vacuum when  $|\phi| = 0$ ) for the non-compact sigma model, a locally flat solution for the compact sigma model and a string-like solution for both types of sigma models. The last one is smooth for the compact scalar manifold.

The plan of this paper is as follows. In section 2 we begin with a review of the  $N = 2$  gauged supergravity model that we consider. In section 3 we give a detailed analysis of the equations that arise from the supersymmetry breaking condition. Section 4 is devoted to the construction of the new supersymmetric solutions mentioned above. Solutions in the flat sigma model are studied in section 5. We conclude in section 6 with some comments and future directions. The supersymmetry breaking condition is worked out in the appendix.

## 2. The model

In this paper we consider  $N = 2$ ,  $U(1)$  gauged supergravity in  $D = 3$  interacting with an arbitrary number of matter multiplets which was constructed in [4]. Its higher dimensional origin is yet to be discovered. The boundary symmetries of this theory were studied in [7] and its extension by including a Fayet-Iliopoulos term was given in [6]. Holographic RG flows in this model were analyzed in [8]. Let us also mention that the model we consider in this paper [4] is a member of a class of theories called abelian Chern-Simons Higgs models coupled to gravity (see [6, 9] and references therein). The field content of the theory is:

- The supergravity multiplet:  $\{e_\mu^a, \tilde{\psi}_\mu, A_\mu\}$
- The scalar multiplet ( $K$  copies):  $\{\phi^\alpha, \tilde{\lambda}^r\}$

All fields except the graviton  $e_\mu^a$  and the gauge field  $A_\mu$  are complex. Here, for the sigma model manifolds we consider  $K = 1$  with the following cases,  $M_+ = S^2 = SU(2)/U(1)$  and  $M_- = H^2 = SU(1,1)/U(1)$ . We define the parameter  $\epsilon = \pm 1$  to indicate the manifolds  $M_\pm$ . The bosonic part of the Lagrangian is <sup>1</sup>

$$\mathcal{L} = \sqrt{-g} \left( \frac{1}{4} R - \frac{1}{16ma^4} \frac{\epsilon^{\mu\nu\rho}}{\sqrt{-g}} A_\mu \partial_\nu A_\rho - \frac{|D_\mu \phi|^2}{a^2(1 + \epsilon|\phi|^2)^2} - V(\phi) \right), \quad (2.1)$$

where  $D_\mu \phi = (\partial_\mu - i\epsilon A_\mu)\phi$  and the potential is given by

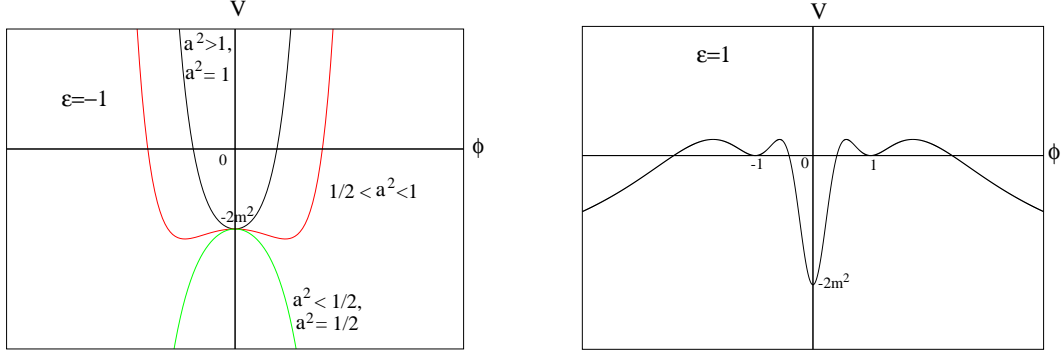
$$V(\phi) = 4m^2 a^2 C^2 \left( |S|^2 - \frac{1}{2a^2} C^2 \right). \quad (2.2)$$

Functions  $C$  and  $S$  are defined as

$$C = \frac{1 - \epsilon|\phi|^2}{1 + \epsilon|\phi|^2}, \quad S = \frac{2\phi}{1 + \epsilon|\phi|^2}. \quad (2.3)$$

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<sup>1</sup>Our conventions are as follows: We take  $\eta_{ab} = (-, +, +)$  and  $\epsilon^{\mu\nu\rho} = \sqrt{-g}\gamma^{\mu\nu\rho}$ . In a coordinate basis a convenient representation for  $\gamma^a$  matrices is  $\gamma_0 = i\sigma^3, \gamma_1 = \sigma^1, \gamma_2 = \sigma^2$  with  $\epsilon^{012} = 1$ . Here 0,1,2 refer to the tangent time, radial and theta directions, respectively, and  $\gamma^2$  is the charge conjugation matrix.



**Figure 1:** The scalar potential  $V$  plotted with respect to  $\phi$ . For  $\epsilon = -1$ ,  $V(|\phi| = 1) \rightarrow \infty$ .

Note that the following algebraic and differential relations hold:

$$|\phi|^2 = \frac{\epsilon(1-C)}{(1+C)}, \quad \epsilon|S|^2 = 1 - C^2, \quad \frac{dC}{d|\phi|} = -\frac{\epsilon|S|^2}{|\phi|}, \quad \frac{d|S|}{d|\phi|} = \frac{C|S|}{|\phi|}. \quad (2.4)$$

The constant “ $a$ ” is the characteristic curvature of  $M_{\pm}$  (e.g.  $2a$  is the inverse radius in the case of  $M_+ = S^2$ ). The gravitational coupling constant  $\kappa$  has been set equal to one and  $-2m^2$  is the  $AdS_3$  cosmological constant. Unlike in a typical AdS supergravity coupled to matter, the constants  $\kappa, a, m$  are not related to each other for non-compact scalar manifolds, while  $a^2$  is quantized in terms of  $\kappa$  in the compact case so that  $\kappa^2/a^2$  is an integer [4]. When  $\epsilon = -1$  for all  $a^2$  there is a supersymmetric AdS vacuum at  $\phi = 0$  and a non-supersymmetric but stable (it satisfies Breitenlohner-Freedman bound [10]) AdS vacuum for  $1/2 < a^2 < 1$ . When  $\epsilon = 1$  there are supersymmetric AdS, Minkowski and non-supersymmetric de Sitter vacua (see figure 1). The forms of the potentials are appropriate for the possibility of existence of topological ( $\epsilon = 1$ ) and non-topological ( $\epsilon = -1$ ) solitons.

The nonlinear scalar covariant derivative  $P_{\mu}$  and the U(1) connection  $Q_{\mu}$  are defined as

$$P_{\mu} = \frac{2\partial_{\mu}\phi}{1 + \epsilon|\phi|^2} - i\epsilon A_{\mu}S, \quad Q_{\mu} = \frac{i\phi \overleftrightarrow{\partial}_{\mu} \phi^*}{1 + \epsilon|\phi|^2} + A_{\mu}C. \quad (2.5)$$

The bosonic field equations that follow from the Lagrangian (2.1) are

$$R_{\mu\nu} = \frac{1}{a^2} P_{(\mu} P_{\nu)}^* + 4Vg_{\mu\nu}, \quad (2.6)$$

$$\epsilon^{\mu\nu\rho} F_{\nu\rho} = -4\epsilon i m a^2 \sqrt{-g} [P^{\mu} S^* - (P^{\mu})^* S], \quad (2.7)$$

$$\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} P_{\nu}) = i\epsilon Q_{\mu} P^{\mu} + 2a^2 (1 + \epsilon|\phi|^2) \frac{\partial V}{\partial \phi^*}. \quad (2.8)$$

The supersymmetric version of the Lagrangian (2.1) is invariant under the following fermionic supersymmetry transformations

$$\delta\tilde{\psi}_{\mu} = \left( \partial_{\mu} + \frac{1}{4} \omega_{\mu}{}^{ab} \gamma_{ab} - \frac{i}{2a^2} Q_{\mu} \right) \varepsilon + m \gamma_{\mu} C^2 \varepsilon, \quad (2.9)$$

$$\delta\tilde{\lambda} = \left( -\frac{1}{2a}\gamma^\mu P_\mu - 2\epsilon ma CS \right) \varepsilon. \quad (2.10)$$

In order to obtain a supersymmetric solution, one needs to solve (2.9) and (2.10) when all fermions are set to zero. It should be noted that the solutions can be naturally divided into two classes:  $a^2 \neq 0$  and  $a^2 = 0$ . (To take the  $a^2 = 0$  limit, certain field rescalings should be performed as explained in section 5.) With these preliminaries at hand, we now look for supersymmetric solutions of this model in the next section.

### 3. Supersymmetry analysis ( $a^2 \neq 0$ )

Our metric ansatz is

$$ds^2 = -F^2 dt^2 + H^2 (G dt + d\theta)^2 + dr^2, \quad (3.1)$$

where  $F, G$  and  $H$  are functions of  $r$  only. We choose the scalar field to be of the form

$$\phi = R(r) e^{in\theta} e^{i\lambda t}, \quad (3.2)$$

where  $n$  and  $\lambda$  are real constants. For the vector field we pick the following gauge

$$A_\mu = (A_t, A_r, A_\theta) = (\psi(r), 0, \chi(r)). \quad (3.3)$$

For a supersymmetric solution, we look for Killing spinors that satisfy  $\delta\tilde{\psi}_\mu = 0$  and  $\delta\tilde{\lambda} = 0$ . For this purpose, we assume a projection of the form  $(1 - \gamma^a b_a)\varepsilon = 0$ , where  $b_a$ 's are some *complex* functions that satisfy  $b^a b_a = 1$ . Since the analysis involves a long calculation, for purposes of readability, we will save those technical details to the appendix and now carry on with the final outcomes of that investigation. After a careful study, one ends up with the following set of equations:

$$(FHZ)' = 4mC^2 FH, \quad (3.4)$$

$$G = g_0 + \frac{FZ}{H}, \quad (3.5)$$

$$\lambda - \epsilon\psi = \frac{1}{C}(\lambda - 2\epsilon a^2 c_2) + qk\epsilon a^2 \frac{H'}{CF} g_0 + 4qk\epsilon ma^2 C, \quad (3.6)$$

$$n - \epsilon\chi = \frac{1}{C}(n - 2\epsilon a^2 c_1) + qk\epsilon a^2 \frac{H'}{CF}, \quad (3.7)$$

$$\frac{R'}{R} = qk \frac{(n - \epsilon\chi)}{FH} + 4\epsilon ma^2 CZ, \quad (3.8)$$

$$\lambda - \epsilon\psi = g_0(n - \epsilon\chi) + 4qk\epsilon ma^2 C. \quad (3.9)$$

Here  $\lambda, k, n, g_0, c_1, c_2$  are arbitrary real constants and  $q^2 = 1$ . Throughout, we use prime to indicate differentiation with respect to  $r$ . Note that (3.9) relates the constants as  $\lambda - 2\epsilon a^2 c_2 = g_0(n - 2\epsilon a^2 c_1)$ . Here we would like to emphasize that for electromagnetic “self-dual” solutions where  $E = -g_0 B$ , we need  $g_0 \neq 0$  and  $k = 0$  as have already been found in [5]. The function  $Z$  is defined as:

$$Z \equiv \frac{p}{F} \sqrt{F^2 - k^2}, \quad p^2 = 1. \quad (3.10)$$

Now one has to check the field equations. It turns out that the scalar field equation (2.8) is identically satisfied as a result of the above set of equations. However, the vector field equation (2.7) yields one new equation

$$a^2 \left( \frac{H'}{F} \right)' + \frac{|S|^2(n - \epsilon\chi)^2}{FH} + 16m^2 a^4 C^2 |S|^2 \frac{H}{F} = 0. \quad (3.11)$$

On the other hand, after some rather lengthy but straightforward calculations, it can be shown that the Einstein field equations (2.6) follow from (3.4), (3.5) and (3.11).

In summary, for a 1/2 supersymmetric solution we need to solve equations (3.8), (3.4) and (3.11) and determine the radial dependences of  $H$ ,  $F$  and  $R$ . (Note that  $(n - \epsilon\chi)$  can be replaced in these equations using (3.7).) Once this is done, the vector field components are determined. After some algebra, it can be shown that these three equations are equivalent to the following set:

$$\frac{|S'|}{|S|} = \frac{\epsilon C \sqrt{16m^2 a^4 C^2 (FH)^2 + W}}{FH}, \quad (3.12)$$

$$\frac{|S'|}{|S|} = \frac{\epsilon a^2 (FH)'}{FH} + \frac{(n - 2\epsilon a^2 c_1) qk}{FH}, \quad (3.13)$$

$$W' = 32\epsilon m^2 a^2 (n - 2\epsilon a^2 c_1) qk C^2 FH, \quad (3.14)$$

where we defined

$$W \equiv k^2(n - \epsilon\chi)^2 + 8\epsilon m a^2 qk C F H Z (n - \epsilon\chi) - 16m^2 a^4 k^2 C^2 H^2. \quad (3.15)$$

When  $k = 0$ , instead of equations (3.14) and (3.15), we have (3.11). Comparing (3.14) and (3.4), we see that

$$W = 8\epsilon m a^2 (n - 2\epsilon a^2 c_1) qk F H Z + w_0, \quad (3.16)$$

where  $w_0$  is a constant that vanishes when  $k = 0$ . Equations (3.15) and (3.16) together give an implicit relationship between the unknown functions  $\chi$ ,  $R$ ,  $F$  and  $H$ . It is interesting to observe that the function  $W$  can be related to a topological invariant as follows:

$$F_{\mu\nu} F^{\mu\nu} = -\frac{32m^2 a^4 |S|^4 W}{F^2 H^2}. \quad (3.17)$$

Note that there is a crucial difference between the  $k = 0$  and  $k \neq 0$  cases in terms of the above invariant; for  $k = 0$ , it vanishes automatically.

With these, our metric becomes:

$$ds^2 = -k^2 dt^2 + 2p F H Z dv dt + H^2 dv^2 + dr^2, \quad v \equiv \theta + g_0 t. \quad (3.18)$$

The curvature scalar of this metric is:

$$g^{\mu\nu} R_{\mu\nu} = -2 \left( \frac{(FH)''}{FH} - 4m^2 C^4 \right). \quad (3.19)$$

The set of equations (3.12), (3.13), (3.14) is quite difficult to analyze in its most general form. This system was also obtained in [6] with  $g_0 = \lambda = c_2 = 0$  and  $k = 1$ , however with the potential modified with the Fayet-Iliopoulos term. Note that the system simplifies when  $k = 0$ . In this case we have  $Z^2 = 1$  and  $W = 0$ , and the equations are completely integrable. This solution was obtained in [5] and it corresponds to a charged, stationary string with gravitational and Chern-Simons waves attached to it [5]. Therefore, we will assume  $k \neq 0$  in this paper. There is another option which simplifies this system, namely  $(n - 2\epsilon a^2 c_1) = 0$  case. Finally, when  $R$  is a constant, that is, the scalar field is just a phase, again the system is completely solvable. Now we analyze these, as well as the general case in detail.

#### 4. Supersymmetric solutions ( $a^2 \neq 0$ )

In this section we will try to solve the set of equations (3.12)-(3.14). We start with some easier subcases and consider the most general case later.

##### 4.1 $(n - 2\epsilon a^2 c_1) = 0$ Case

In this case, we see from (3.16) that

$$W = w_0, \tag{4.1}$$

where  $w_0$  is an arbitrary real constant. Then we find from (3.13) that

$$FH = f_0 |S|^{\epsilon/a^2}, \tag{4.2}$$

which makes (3.12) a separable first order differential equation. Now we proceed with an investigation of the  $w_0 = 0$  and  $w_0 \neq 0$  cases separately.

##### 4.1.1 $w_0 = 0$

When  $w_0 = 0$ , (3.12) is easily integrated [5], but its explicit form will not be necessary for the discussion below. From (3.4), one finds

$$Z = 1 - \frac{u_0}{FH}, \tag{4.3}$$

where  $u_0$  is a real constant. Using (4.3) and (4.2), one obtains

$$H^2 = \frac{f_0^2}{k^2} (2u_0 f_0 |S|^{\epsilon/a^2} - u_0^2). \tag{4.4}$$

For the vector field (3.7), one gets

$$n - \epsilon\chi = \frac{4\epsilon m a^2 u_0 f_0}{qk} C, \tag{4.5}$$

which is smooth for  $\epsilon = 1$ . The metric (3.18) now becomes

$$ds^2 = -k^2 du^2 + 2f_0 |S|^{\epsilon/a^2} dudv + dr^2, \quad u \equiv pt + \frac{u_0 f_0}{k^2} v, \tag{4.6}$$

which reduces to the string solutions (with no waves attached) that were found in [5] when  $k = 0$ . The scalar field in [5] and the solution presented here is the same; however,  $(n - \epsilon\chi)$  in that case was proportional to  $1/C$  from (3.7). This change makes the vector field smooth for  $\epsilon = 1$ . Unfortunately, we couldn't identify what this solution represents. However, because of the fact that this and the string solution presented in [5] both have the same curvature invariants and the same scalar field, we call this as a 'string-like' solution. Note that, for both solutions the  $F^2$  invariant vanishes (3.17). The main difference of this new one is the absence of the  $SO(1,1)$  worldsheet symmetry of the string solution [5].

There is a curvature singularity (3.19) as  $C^2 \rightarrow \infty$  which can be seen from the curvature invariants

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} &= -8m^2 C^2 (3C^2 - 8a^2 |S|^2), \\ R^{\mu\nu} R_{\mu\nu} &= 64m^4 C^4 (3C^4 - 16a^2 C^2 |S|^2 + 24a^4 |S|^4). \end{aligned} \quad (4.7)$$

When  $\epsilon = -1$ , by performing a coordinate transformation  $\rho = 1/|S|^2$  ( $0 \leq \rho < \infty$ ), the metric (4.6) becomes

$$ds^2 = -k^2 du^2 + 2f_0 \rho^{1/(2a^2)} dudv + \frac{d\rho^2}{64m^2 a^4 (\rho + 1)^2}, \quad \epsilon = -1. \quad (4.8)$$

By inspection, it is seen that there is no horizon and we have a naked singularity at  $\rho = 0$  (or  $C^2 \rightarrow \infty$ ). As  $\rho \rightarrow \infty$  (or  $C^2 \rightarrow 1$ ) the solution becomes locally  $AdS_3$  whose metric corresponds to a generalized Kaigorodov metric [11].

Let us now consider the  $\epsilon = 1$  case in more detail. We first define a new radial coordinate  $\rho = M/C^2$  ( $M \leq \rho < \infty$ ), where  $M$  is a positive constant. Then (4.6) becomes

$$ds^2 = -k^2 du^2 + 2f_0 \left(1 - \frac{M}{\rho}\right)^{1/(2a^2)} dudv + \frac{d\rho^2}{64m^2 a^4 (\rho - M)^2}, \quad \epsilon = 1. \quad (4.9)$$

We see that there is a horizon as  $\rho \rightarrow M$ ; from the curvature invariants (4.7) the local geometry is observed to be locally  $AdS_3$ , which has a Kaigorodov [11] type of structure. As  $\rho \rightarrow \infty$ , the solution is asymptotically flat. Since  $|\phi|^2 = (\sqrt{\rho} - \sqrt{M})/(\sqrt{\rho} + \sqrt{M})$ , the scalar field and the vector field (4.5) are smooth everywhere. Both fields have asymptotic values that are expected from a topological soliton, however the presence of a horizon prevents us from labeling this solution as such.

Now let us look at the behavior of the geodesics. The geodesic equation associated with the metric (4.9) is:

$$\frac{1}{64m^2 a^4} \left(\frac{\dot{\rho}}{\rho}\right)^2 = \alpha \left(1 - \frac{M}{\rho}\right)^2 - \frac{EP}{2f_0} \left(1 - \frac{M}{\rho}\right)^{2 - \frac{1}{2a^2}} - \frac{P^2 k^2}{4f_0^2} \left(1 - \frac{M}{\rho}\right)^{2 - \frac{1}{a^2}}, \quad (4.10)$$

where the dot denotes derivative with respect to an affine parameter and  $\alpha = 0$  or  $\alpha = -1$  for null or timelike geodesics, respectively. In this equation  $E$  and  $P$  are the conserved quantities associated with the flow of the tangent vector of a geodesic corresponding to the  $t$  and  $v$  variables. It is easy to see that timelike geodesics can not reach the horizon since there is always a turning point. For null geodesics, if one demands the geodesics to reach the



$\rho \rightarrow \infty$  limit, one requires  $-k^2 P^2 - 2EPf_0 > 0$ . One can show that in this case there is no turning point and such geodesics reach the horizon. However, for  $1/a^2 = 1, 3 \pmod{4}$ , these geodesics can not extend beyond the horizon since (4.10) becomes imaginary. Moreover, for  $1/a^2 = 2 \pmod{4}$ , it is easy to see that the geodesics do not cross the horizon since the right hand side of (4.10) becomes negative then. Therefore, the solution is well-defined for all values of  $1/a^2$  except when  $1/a^2 = 0 \pmod{4}$ .

#### 4.1.2 $w_0 \neq 0$ and $\epsilon = -1$

For general  $a^2$ , when  $w_0 \neq 0$ , the integration of (3.12) is quite complicated. Therefore we will mainly concentrate on the  $a^2 = 1$  case. In this case, we first introduce a new constant  $|b| \leq 1$  such that  $w_0 = -16m^2 b^2$ , to simplify the discussion. Now (3.12) implies that

$$\frac{R'}{R} = 4m\sqrt{C^2 - b^2|S|^2}.$$

From this it follows that

$$dr = \frac{dC}{4m|S|^2\sqrt{C^2 - b^2|S|^2}}.$$

Furthermore, introducing  $U \equiv FHZ$  and taking  $U = U(C)$ , one finds from (3.4) that

$$\frac{dU}{dC} = f_0 \frac{C^2}{(C^2 - 1)^{3/2}\sqrt{C^2 - b^2(C^2 - 1)}},$$

which can be integrated using an elliptic function  $E^2$  as

$$U(C) = bf_0 \frac{-C\sqrt{1 + (-1 + \frac{1}{b^2})C^2} + \sqrt{1 - C^2}E(\arcsin C|1 - \frac{1}{b^2})}{\sqrt{C^2 - 1}}.$$

At first sight, this seems to be complex valued, however it can be verified that  $U(C)$  is always real for  $C > 1$  (which is automatically satisfied for  $\epsilon = -1$ ) and  $|b| < 1$ . With these, the metric can be cast in the form

$$ds^2 = -(pkt - U(C)dv)^2 + f_0^2 \frac{dv^2}{|S|^2} + \frac{dC^2}{16m^2|S|^4(C^2 - b^2|S|^2)}.$$

Note that  $C^2 - b^2|S|^2 = 0$  when  $C^2 = b^2/(b^2 - 1) < 1$ , which is not allowed and therefore there is no horizon! The curvature invariants for this metric are

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} &= -8m^2(8C^2 - 5C^4 + 4b^2|S|^4), \\ R^{\mu\nu} R_{\mu\nu} &= 64m^4[8b^4|S|^8 - 8b^2C^2|S|^4(2|S|^2 - 1) + C^4(24 - 32C^2 + 11C^4)], \end{aligned}$$

from which it follows that as  $C \rightarrow 1$  ( $|S| \rightarrow 0$ ), the solution becomes locally AdS. The only place where a curvature singularity may appear is at  $C \rightarrow \infty$  (and thus  $|S| \rightarrow \infty$ ). However, this may not always be an allowed limit. To see this, let us consider the  $b = 1$

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<sup>2</sup>The elliptic function of the 2nd kind is defined as  $E(\phi|m) = \int_0^\phi (1 - m \sin^2 \theta)^{1/2} d\theta$ ,  $\phi \in (-\pi/2, \pi/2)$ .

case which simplifies the calculations above. Then (3.12) becomes  $R'/R = -4m$ . Defining a new radial coordinate  $\rho = 1/|S|^2$ , we obtain

$$FHZ = f_0\sqrt{\rho+1} - \frac{f_0}{2} \ln \left( \frac{\sqrt{\rho+1}+1}{\sqrt{\rho+1}-1} \right). \quad (4.11)$$

From (4.2), we have  $FH = f_0\sqrt{\rho}$  which yields,

$$Z = \sqrt{\frac{\rho+1}{\rho}} - \frac{1}{2\sqrt{\rho}} \ln \left( \frac{\sqrt{\rho+1}+1}{\sqrt{\rho+1}-1} \right). \quad (4.12)$$

Since we have  $|Z| \leq 1$  from (3.10), we see that this forbids  $\rho$  to reach 0.

In terms of the new radial coordinate,  $R = -\sqrt{\rho} + \sqrt{1+\rho}$ , which clearly shows that the scalar field is always smooth and finite. From (3.8), we obtain

$$\chi = -n + \frac{4mf_0}{qk} \left[ \frac{1}{\sqrt{\rho}} - \sqrt{\frac{\rho+1}{\rho}} \ln \left( \frac{\sqrt{\rho+1}+1}{\sqrt{\rho+1}-1} \right) \right], \quad (4.13)$$

which is again smooth everywhere. This also implies the smoothness of  $\psi$  from (3.9). All these matter fields have the expected behavior of a non-topological soliton as  $\rho \rightarrow \infty$ .

### 4.1.3 $w_0 \neq 0$ and $\epsilon = 1$

When  $a^2 = 1$ , the discussion is analogous to the one for the  $\epsilon = -1$  case: We start by introducing a new constant  $b > 0$  such that  $w_0 = 16m^2b^2$ . In this case (3.12) yields

$$\frac{R'}{R} = 4m\sqrt{C^2 + \frac{b^2}{|S|^2}}.$$

We record here in passing that now

$$dr = \frac{dC}{4m|S|^2\sqrt{C^2 + \frac{b^2}{|S|^2}}}.$$

Introducing  $U \equiv FHZ$  as before and taking  $U = U(C)$  again, one obtains from (3.4) that

$$\frac{dU}{dC} = f_0 \frac{C^2}{\sqrt{C^2(1-C^2) + b^2}},$$

whose integration yields the following complicated expression in terms of elliptic functions  $E$  and  $F$ <sup>3</sup>:

$$U(C) = \frac{if_0(\xi+1)\sqrt{\xi-1}}{2\sqrt{2(C^2(1-C^2)+b^2)}} \sqrt{1 + \frac{2C^2}{\xi-1}} \sqrt{1 - \frac{2C^2}{\xi+1}} \times \\ \left( E \left( i \operatorname{arcsinh} \left[ \sqrt{\frac{2}{\xi-1}} C \right] \middle| \frac{1-\xi}{1+\xi} \right) - F \left( i \operatorname{arcsinh} \left[ \sqrt{\frac{2}{\xi-1}} C \right] \middle| \frac{1-\xi}{1+\xi} \right) \right),$$

<sup>3</sup>The elliptic function of the 1st kind is given by  $F(\phi|m) = \int_0^\phi (1 - m \sin^2 \theta)^{-1/2} d\theta$ ,  $\phi \in (-\pi/2, \pi/2)$ .

where we have used  $\xi \equiv \sqrt{1+4b^2} > 1$  for convenience. This may again seem to be complex valued, however it can be verified that  $U(C)$  is always real for  $0 < C < 1$  (which is automatically satisfied for  $\epsilon = 1$ ). The metric now reads

$$ds^2 = -(pkdt - U(C)dv)^2 + f_0^2|S|^2dv^2 + \frac{dC^2}{16m^2|S|^2(C^2|S|^2 + b^2)}.$$

It is clear that there is no horizon. The curvature invariants for this metric are

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} &= 8m^2(4b^2 - 11C^4 + 8C^2), \\ R^{\mu\nu} R_{\mu\nu} &= 64m^4(8b^4 - 8b^2C^2(4C^2 - 3) + C^4(24 - 64C^2 + 43C^4)), \end{aligned}$$

which are both regular for  $0 < C < 1$ .

Looking at the curvature scalar above, one notices that this solution approaches neither to AdS ( $C = 1$  limit, where  $g^{\mu\nu} R_{\mu\nu} = -24m^2$ ) nor the Minkowski vacuum ( $C = 0$  limit). Indeed, if one plots  $(1 - Z^2) = k^2/F^2$ , one finds that this becomes negative before the  $C = 1$  point is reached. We thus conclude that this solution must be ruled out.

#### 4.2 $R' = 0$ Case

For the sake of completeness, we also consider this case. Looking at the supersymmetric vacua in figure 1, we see that one should set  $R = 0$  when  $\epsilon = -1$ , and  $R = 0$  or  $R = 1$  when  $\epsilon = 1$ . The scalar field is just a pure phase now. The functions  $C$  and  $|S|$  are just constants and by defining

$$\alpha \equiv -\frac{(n - 2\epsilon a^2 c_1)qk\epsilon}{a^2} \quad \text{and} \quad x_0 \equiv -4m \left( \frac{1 - \epsilon R_0^2}{1 + \epsilon R_0^2} \right)^2,$$

where  $|\phi| = R_0$ , one finds from (3.13) and (3.4) that

$$FH = \alpha r + \beta \quad \text{and} \quad FHZ \equiv U(r) = -x_0 \left( \alpha \frac{r^2}{2} + \beta r + u_0 \right),$$

for some integration constants  $\beta$  and  $u_0$ . The metric can now be cast in the form

$$ds^2 = -(pkdt - U(r)dv)^2 + (\alpha r + \beta)^2 dv^2 + dr^2, \tag{4.14}$$

whose curvature invariants simply read

$$g^{\mu\nu} R_{\mu\nu} = \frac{x_0^2}{2} \quad \text{and} \quad R^{\mu\nu} R_{\mu\nu} = \frac{3}{4}x_0^2,$$

which indicate that the metric describes a local  $dS$  spacetime, when  $x_0 \neq 0$ . This enforces us to set  $x_0 = 0$ , i.e.  $R_0 = 1$ , which is only allowed for  $\epsilon = 1$ . This also sets  $Z = 0$ , since now  $U(r) = 0$ . These imply from (3.7) that  $\chi = \epsilon n$ . The metric (4.14) becomes (after absorbing constants in the metric by redefining the  $t$  and the  $v$  coordinates)

$$ds^2 = -dt^2 + r^2 dv^2 + dr^2, \tag{4.15}$$

which is a locally flat spacetime with Euclidean Rindler spatial sections.

### 4.3 General Case

To analyze the general case (i.e.  $(n - 2\epsilon a^2 c_1) \neq 0$  and  $k \neq 0$ ), we first note that from equations (3.4)-(3.9), one also finds

$$\left[ FH \frac{R'}{R} \right]' = 16\epsilon m^2 a^2 CFH(C^2 - a^2|S|^2). \quad (4.16)$$

Defining a new function  $Y$  through the relation

$$FH = f_0 |S|^{\epsilon/a^2} Y, \quad (4.17)$$

one can also show that

$$\frac{Y'}{Y} = -\frac{qk\epsilon(n - 2\epsilon a^2 c_1)}{a^2 FH}. \quad (4.18)$$

Using (4.18), we can write (4.16) as

$$\frac{1}{Y} \frac{\partial}{\partial Y} \left[ Y \frac{\partial \ln R}{\partial Y} \right] = -\frac{\partial V_{\text{eff}}}{\partial (\ln R)} = \frac{16\epsilon m^2 a^6 f_0^2}{(n - 2\epsilon a^2 c_1)^2 k^2} C |S|^{2\epsilon/a^2} (C^2 - a^2|S|^2), \quad (4.19)$$

which decouples  $R$  from other unknown metric functions as was observed in [6]. A simple integration yields the effective potential to be

$$V_{\text{eff}} = -\frac{8m^2 a^8 f_0^2}{(n - 2\epsilon a^2 c_1)^2 k^2 (\epsilon + a^2)} C |S|^{2\epsilon/a^2} [\epsilon C^2 + a^2(C^2 - \epsilon|S|^2)]. \quad (4.20)$$

These can be interpreted as describing a classical mechanical system where a (fictitious) point particle is subject to a motion due to an effective potential.

When  $\epsilon = 1$ , it was shown in [6] that there is no vortex solution where the scalar field approaches to a Minkowski vacuum as  $r \rightarrow \infty$  and to an AdS vacuum as  $r \rightarrow 0$ . To see this in our setup, we impose the following AdS behavior around  $r = 0$ :  $H \approx r$ ,  $F \approx k$ . Then the regularity of the scalar field implies that  $q = 1$ ,  $c_1 = 1/2$ , and we get  $\chi = 0$  and  $R = R_0 r^n$ , where  $R_0$  is a constant. At the other end, as  $r \rightarrow \infty$  demanding  $R \approx 1$ ,  $H \approx H_\infty r$ ,  $FHZ = \text{const.}$ , the regularity of the vector field  $\chi$  (3.7) imposes that  $H_\infty = 1 - n/a^2$ . A well defined conical geometry requires  $n/a^2 < 1$ . However, this is not possible due to the fact that  $1/a^2$  is an integer for  $\epsilon = 1$ . (When  $n = 0$ , one is forced to set  $|\phi| = 1$  [12] which is analyzed in section 4.2.)

For  $\epsilon = -1$ , there may be a non-topological soliton, however (4.19) is hard to analyze. In principle, one has to study (4.19) in the limits  $C \rightarrow 1$  and  $C \rightarrow \infty$  separately and match the two solutions in a unique fashion. However, the limit  $C \rightarrow 1$  (i.e.  $R \rightarrow 0$ ) is quite difficult to work with due to the divergent right hand side (the exponent of  $|S|$  is negative), however, techniques developed in [12] might be applicable. If such a solution exists, then  $C \rightarrow \infty$  shouldn't be accessible since this would make the vector field component  $\psi$  divergent (3.9).

### 5. Flat sigma model ( $a^2 = 0$ )

To take the  $a^2 = 0$  limit in our model, first one has to rescale  $A_\mu \rightarrow a^2 A_\mu$  and  $\phi \rightarrow a\phi$ . Then we have  $C \rightarrow 1$ ,  $S \rightarrow 2a\phi$ , and one obtains  $N = 2$ ,  $AdS_3$  supergravity with cosmological

constant  $-2m^2$  coupled to an  $R^2$  sigma manifold [4]. This coincides with the flat sigma model limit of the  $N = 2$  theory discussed in [13] as was shown in [7]. The Lagrangian (2.1) now becomes

$$\mathcal{L} = \sqrt{-g} \left( \frac{1}{4}R - \frac{1}{16m} \frac{\epsilon^{\mu\nu\rho}}{\sqrt{-g}} A_\mu \partial_\nu A_\rho - |\partial_\mu \phi|^2 + 2m^2 \right), \quad (5.1)$$

and its fermionic supersymmetry transformations are

$$\delta \tilde{\psi}_\mu = \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} - \frac{i}{2} [i\phi \overleftrightarrow{\partial}_\mu \phi^* + A_\mu] \right) \varepsilon + m \gamma_\mu \varepsilon, \quad (5.2)$$

$$\delta \tilde{\lambda} = -(\gamma^\mu \partial_\mu \phi) \varepsilon. \quad (5.3)$$

To find a 1/2 supersymmetric solution, we again choose the same metric ansatz (3.1) and use the same form of scalar and vector fields given in (3.2) and (3.3). Now using the projection condition (A.3) in  $\delta \tilde{\lambda} = 0$  and  $\delta \tilde{\psi}_\mu = 0$ , we find

$$R = R_0 = \text{const.}, \quad (5.4)$$

$$G = \frac{\lambda}{n} + \frac{FZ}{H}, \quad (5.5)$$

$$\chi = 2c_1 - 2nR_0^2 \pm \frac{kH'}{F}, \quad (5.6)$$

$$\psi = 2c_2 - 2\lambda R_0^2 \pm 4km \pm \frac{kH' \lambda}{F n}. \quad (5.7)$$

The remaining unknown functions  $F$  and  $H$  are to be determined from

$$(FHZ)' = 4mFH, \quad (5.8)$$

$$\left( \frac{H'}{F} \right)' = -\frac{4R_0^2 n^2}{FH}. \quad (5.9)$$

Here the first equation (5.8) comes from the supersymmetry analysis and the second one (5.9) follows from the field equations. When  $k = 0$ , we have  $Z^2 = 1$  (3.10) and (5.9) is easily integrable then. This case was studied in [5]. The metric in this case takes the form

$$ds^2 = -2pf_0 e^{-4pmr} dv dt + (h_0 e^{-4pmr} + h_1 + h_2 r) dv^2 + dr^2, \quad (5.10)$$

where  $f_0$  is an integration constant. Here  $h_0$  and  $h_1 = (pmc_0 + 2R_0^2 n^2)/4m^2$  (or  $c_0$ ) are arbitrary real constants and we have  $h_2 = -2R_0^2 n^2 p/m$ .

A study of the curvature invariants indicate that the solution has constant negative curvature  $-24m^2$  and it is locally  $AdS_3$  [5]. When  $h_1 = h_2 = 0$ , the metric is the  $AdS_3$  metric in Poincaré coordinates. The  $h_1$  term can be obtained by using the Garfinkle-Vachaspati method [14, 15] and it describes a wave in  $AdS_3$ . Actually the metric with  $h_2 = 0$  has already been discussed in [16] and it corresponds to a generalized Kaigorodov metric [11]. Its equivalence to the extreme BTZ black hole [17] can be shown [16, 18]. To see this, first scale the  $v$  and the  $t$  coordinates such that the constants  $f_0$  and  $h_0$  are set to one, and next define  $H = \rho$  as the new radial coordinate in (5.10). Then we get

$$ds^2 = -(4m^2 \rho^2 - 2mh_1) dt^2 + h_1 d\beta dt + \rho^2 d\beta^2 + \frac{4\rho^2}{(4m\rho^2 - h_1)^2} d\rho^2, \quad \beta \equiv v - pt, \quad (5.11)$$

where we have also performed the  $t \rightarrow (\sqrt{m}/p)t$ ,  $\rho \rightarrow 2\sqrt{m}\rho$  and  $\beta \rightarrow [1/(2\sqrt{m})]\beta$  scalings. Now (5.11) is the extreme BTZ metric with total angular momentum  $J = h_1$  and the total mass  $M = 2mJ = 2mh_1$ .

When  $h_2 \neq 0$ , this spacetime is another pp-wave in  $AdS_3$ . It is clear that it exists only for a non-zero scalar field. In this case, if we perform the coordinate change  $H = \rho$  in (5.10), then when the  $h_2$  term is negligible, the extreme BTZ metric (5.11) receives corrections involving  $(\ln \rho)$  terms which prevents the existence of a horizon. This signifies that this solution might be interpreted as a self-gravitating soliton [13].

Another complete solution for (5.8) and (5.9) is found when

$$\frac{H'}{F} = h_0, \tag{5.12}$$

where  $h_0$  is a constant. Defining a new radial coordinate  $H = \rho$ , one then finds supersymmetric solutions that were already studied in [13]. In this case, the vector field components turn out to be constants. Unfortunately, we couldn't solve (5.8) and (5.9) in their full generality.

## 6. Conclusions

We would now like to make some remarks about our results and suggest some future directions. In this paper, we have found new supersymmetric solutions in addition to the already known ones. For the non-compact sigma model, we gave an explicit solution which we interpret as a non-topological soliton. To our knowledge, this is the first example of such an exact solution for a nonlinear sigma model coupled to supergravity. In addition, we also found a locally flat solution for the compact sigma model and a solution that we termed 'string-like' for both types of sigma models. As we discussed above, these 'string-like' solutions are obtained when  $w_0 = 0$  and they resemble the string solution (without waves) given in [5]. The differences lie in an extra term in the metric which doesn't affect the curvature invariants and the form of the vector field. This suggests a relationship between these two solutions which would be interesting to understand.

We observed that in the flat sigma model limit ( $a^2 \rightarrow 0$ ) of our theory, the BTZ [17] black hole solution arises [13]. However for  $a^2 \neq 0$ , we weren't able to find a solution which we could label as a black hole. This type of solution may not be allowed when the nonlinear sigma model scalars are active. It is desirable to see whether this is true, by finding out all possible supersymmetric solutions as was done recently in some higher dimensional models (for a review see [19].) One further generalization that is worth studying is to allow the coupling of more than one matter multiplets to supergravity.

In all our solutions AdS space emerged when we took certain limits, which makes these useful for studying the  $AdS/CFT$  duality. Half supersymmetric, smooth solutions recently attracted much attention after the appearance of [20] where it was shown that such solutions correspond to droplets in phase space occupied by the fermions on the  $CFT$  side. Unfortunately, the  $CFT$  dual of the model that we studied is not known yet. Once that is established, our explicit soliton solution might be important from this perspective.

We hope that our results will be useful in studying supersymmetric solutions of more complicated supergravity theories which are coupled to sigma model systems in higher dimensions, as well as in  $D = 3$  (for a review see [21]). It is especially attractive to study the  $D = 3$ ,  $N = 8$  model [22], since its higher dimensional origin and its dual *CFT* are well-known.

Another thing to consider would be the explicit calculation of the conserved charges for the non-topological soliton that we found. It would be interesting to investigate the energy bound [13, 23] for this soliton. We hope to report on these issues soon.

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### A. Analysis of the supersymmetry breaking condition

In this appendix, we give the technical details that lead to (3.4)-(3.9). We note that our analysis of  $\delta\tilde{\psi}_\mu = 0$  goes along the lines of [13].

By defining  $B_\mu = \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + mC^2\gamma_\mu$ , one can write the Killing spinor equation (2.9) as

$$D_\mu\varepsilon \equiv \left( \partial_\mu + B_\mu - \frac{i}{2a^2}Q_\mu \right) \varepsilon = 0. \tag{A.1}$$

Now imposing  $[D_\mu, D_\nu]\varepsilon = 0$ , it can be shown that this integrability condition is equivalent to

$$(S^\sigma{}_c\gamma^c + G^\sigma)\varepsilon = 0, \tag{A.2}$$

where we have defined

$$S^\sigma{}_c \equiv \epsilon^{\mu\nu\sigma}(\partial_\mu B_{\nu c} + \epsilon_{abc}B_\mu{}^a B_\nu{}^b) \quad \text{and} \quad G^\sigma \equiv -\frac{i}{2a^2}\epsilon^{\mu\nu\sigma}\partial_\mu Q_\nu$$

for simplicity. For a 1/2 supersymmetric solution, we now assume a projection of the form

$$(1 - \gamma^a b_a)\varepsilon = 0, \tag{A.3}$$

where  $b_a$ 's are some *complex* functions that satisfy  $b^a b_a = 1$ . This condition can easily be solved with a spinor of the form

$$\varepsilon = N(1 + \gamma^a b_a)\varepsilon_0, \tag{A.4}$$

where  $N$  is an arbitrary complex function and  $\varepsilon_0$  is a constant spinor. Inserting this solution for the spinor into the integrability condition (A.2), one finds

$$S^\sigma{}_c + \epsilon_{abc}S^{\sigma a}b^b - S^\sigma{}_a b^a b_c = 0. \tag{A.5}$$

One now calculates the components of  $S^{\sigma a}$  and  $B_{\mu}{}^a$  using the ansatz (3.1) for the metric and substitutes these to the condition (A.5) which yields

$$\begin{aligned} \frac{B_{t2}}{B_{\theta 2}} &= \frac{B_{t0}}{B_{\theta 0}} = \Lambda(r) \quad \text{for some function } \Lambda(r), \\ S^{t2}(b_1 - b_2 b_0) - S^{t0}(1 + (b_0)^2) &= 0, \\ S^{\theta 2}(b_1 - b_2 b_0) - S^{\theta 0}(1 + (b_0)^2) &= 0. \end{aligned} \tag{A.6}$$

On the other hand, going back to the original Killing spinor equation (A.1) and using (A.4) leads to

$$\partial_{\mu}(\ln N) - \frac{i}{2a^2} Q_{\mu} + B_{\mu c} b^c = 0, \tag{A.7}$$

$$\partial_{\mu} b_a + B_{\mu a} - B_{\mu c} b^c b_a - \epsilon_{abc} b^b B_{\mu}{}^c = 0. \tag{A.8}$$

However, (A.6) together with the  $\mu = t$  and  $\mu = \theta$  components of (A.8) imply that

$$(\partial_t - \Lambda(r)\partial_{\theta})b_a = 0. \tag{A.9}$$

Now taking the  $r$  derivative of (A.9) and using the information inherent in the  $\mu = r$  component of (A.8) implies that  $\partial_{\theta} b_a = 0$ , and thus  $\partial_r b_a = 0$ , unless  $\Lambda'(r) = 0$ . In what follows we assume that  $b_a = b_a(r)$  only and keep  $\Lambda(r)$  arbitrary so that one is led to

$$b_0 = b_2 Z \pm \sqrt{Z^2 - 1}, \quad b_1 = -Z \mp b_2 \sqrt{Z^2 - 1},$$

where

$$Z(r) \equiv \frac{B_{\theta 2}}{B_{\theta 0}} = -\frac{1}{2} \frac{H^2 G'}{F H'} + 2mC^2 \frac{H}{H'} \tag{A.10}$$

for our choice of metric functions (3.1). Using these with the  $\mu = r$  component of (A.8), one also finds that

$$b'_2 = \pm B_{r1}(r)(1 - (b_2)^2)\sqrt{Z^2 - 1} \quad \text{and} \quad b'_1 = B_{r1}(r)((b_1)^2 - 1),$$

which lead to

$$Z' + 2B_{r1}(r)(Z^2 - 1) = 0. \tag{A.11}$$

Now using the explicit components of  $B_{\mu a}$  in (A.6), one also obtains

$$B_{r1}(r) = \frac{F'}{2FZ}, \tag{A.12}$$

which finally yields (3.10) thanks to (A.11).

For the metric ansatz employed, the explicit form of  $B_{r1}(r)$  is

$$B_{r1}(r) = \frac{1}{4} G' \frac{H}{F} + mC^2.$$

Using this in conjunction with (A.12) and (A.10) leads to (3.4) and (3.5), together with

$$\Lambda(r) = g_0 + 4mC^2 \frac{F}{H'} \quad \text{where } g_0 \text{ is an integration constant,}$$



after a careful scrutiny. All of these can be used in the  $\mu = r$  component of (A.8) to obtain the complex functions  $b_a$  as

$$\begin{aligned}
 b_2 &= \frac{\beta \left( \frac{2}{F}(-ik + \sqrt{F^2 - k^2}) \right)^p - 1}{\beta \left( \frac{2}{F}(-ik + \sqrt{F^2 - k^2}) \right)^p + 1}, \\
 b_0 &= \pm i \frac{k}{F} + \frac{p}{F} b_2 \sqrt{F^2 - k^2}, \quad \text{where } p^2 = 1, \\
 b_1 &= -\frac{p}{F} \sqrt{F^2 - k^2} \mp i \frac{k}{F} b_2,
 \end{aligned} \tag{A.13}$$

for some integration constant  $\beta$ .

Going back to (A.7), one can now determine the function  $N$  in (A.4) as

$$\ln N = \frac{1}{2} \ln F \pm ip \frac{k}{2} I(F(r)) + \tilde{n}(\theta, t),$$

where  $\tilde{n}(\theta, t)$  is an arbitrary function to be determined and

$$I(F(r)) = \begin{cases} \frac{1}{2k} \left\{ 2 \arctan \left[ \frac{4\beta k}{(4\beta^2 - 1)F} \right] + i \left( \ln \left[ \frac{F^2}{(1 - 4\beta^2)^2 F^2 + 16\beta^2 k^2} \right] \right. \right. \\ \left. \left. - 2 \ln \left[ \frac{2(4\beta^2 - 1)[4\beta F + ik + \sqrt{F^2 - k^2} + 4\beta^2(-ik + \sqrt{F^2 - k^2})]}{(1 + 4\beta^2)^2 ((4\beta^2 - 1)F + 4i\beta k)} \right] \right) \right\}, & p = +1 \\ \frac{1}{2k} \left\{ 2 \arctan \left[ \frac{4\beta k}{(\beta^2 - 4)F} \right] - i \left( \ln \left[ \frac{F^2}{(\beta^2 - 4)^2 F^2 + 16\beta^2 k^2} \right] \right. \right. \\ \left. \left. + 2 \ln \left[ \frac{2(\beta^2 - 4)[-4\beta F + \beta^2(-ik + \sqrt{F^2 - k^2}) + 4(ik + \sqrt{F^2 - k^2})]}{(\beta^2 + 4)^2 ((\beta^2 - 4)F - 4i\beta k)} \right] \right) \right\}, & p = -1 \end{cases}. \tag{A.14}$$

However, using the  $\mu = t$  and  $\mu = \theta$  components of (A.7), one can show that  $\tilde{n}(\theta, t)$  necessarily has the form  $\tilde{n}(\theta, t) = i(c_1\theta + c_2t)$  for some constants  $c_1$  and  $c_2$ .

All of the steps taken so far has been verified to be consistent in themselves. Putting things together, the Killing spinor is finally obtained as

$$\varepsilon = \sqrt{F(r)} e^{i(c_1\theta + c_2t)} e^{ipqkI(F(r))/2} (1 + \gamma^a b_a) \varepsilon_0, \tag{A.15}$$

where  $b_a$ 's and the function  $I(F(r))$  are given in (A.13) and (A.14), respectively. Furthermore, the constants  $c_2$  and  $c_1$  can also be used in finding the components of the vector field  $A_\mu$  (3.3), which yield (3.6) and (3.7), respectively.

One is now left with the other supersymmetry transformation  $\delta\tilde{\lambda} = 0$  (2.10). It can easily be verified that this can be written in the form

$$(\Xi^a \gamma_a + \Xi) \varepsilon = 0,$$

where

$$\Xi^0 = \frac{i}{F} ((\lambda - \epsilon\psi) - G(n - \epsilon\chi)), \quad \Xi^1 = \frac{R'}{R}, \quad \Xi^2 = \frac{i}{H} (n - \epsilon\chi) \quad \text{and} \quad \Xi = 4\epsilon m a^2 C.$$

However, one easily finds that the projection condition used for the Killing spinor (A.3) implies that  $\Xi^a b_a + \Xi = 0$  as well. Using  $b^a b_a = 1$  and the standard  $\gamma_a$  algebra, one also finds that  $\Xi^a$  are related by

$$\Xi b_a + \Xi_a + \epsilon_{abc} \Xi^b b^c = 0.$$

A careful study of these equations finally shows that there are in fact only two conditions that are imposed by (2.10) on the metric and matter field components, which are simply (3.8) and (3.9).

This concludes our analysis for the simultaneous vanishing of the supersymmetry transformations (2.9) and (2.10) for our ansatz (3.1), (3.2) and (3.3), and the derivation of the equations (3.4) through (3.9).

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